

**B.C.A. (Pt.-II)**

Disc. Math.

**202/232**

**B.C.A. (Part-II) Examination, 2023** 401258

(Faculty of Science)

(Three Year Scheme of 10+2+3 Pattern)

**Discrete Mathematics**

**Paper : 202/232**

**Time Allowed : 3 Hours**

**Maximum Marks : 100**

Answer of all the questions (Short answer as well as are to be given in the main answer-book only. Answers of short answer type questions must be given in sequential order. Similarly all the parts of one question of descriptive part should be answered at one place in the answer-book. One complete question should not be answered at different places in the answer-book.

Write your roll number on question paper before you start writing answers of questions.

Question paper consists of **Three** parts.

**All Three** parts are **Compulsory**

**PART-I :** (Very short answer) consists of 10 questions of 2 marks each. Maximum limit for each question is up to 40 words.

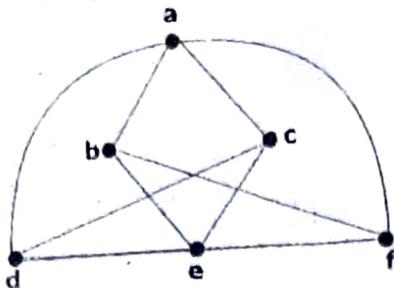
**PART-II :**(Short answer) consists of 5 questions of 4 marks each, Maximum limit for each question is up to 80 words.

**PART-III :** (Long answer) consists of 5 questions of 12 marks each with one question from each unit with internal choices.

**PART-I**

1. Attempt all the parts of the questions : 10×2=20
- (a) Convert  $(1101101.011)_2$  into  $( )_{10}$ .
  - (b) Computer  $(436)_{10} + (51)_{10} = ( )_2$ .
  - (c) If  $A = \{2,3,4\}$  and  $B = \{3,4,5,6\}$  then find the symmetric difference of the sets A and B.
  - (d) Define Reflexive relation with example.
  - (e) Show that  $(p \wedge q) \Rightarrow p$  is a tautology.
  - (f) What is Universal Gate? Name the types of Universal Gates.
  - (g) What is Bipartite Graph? Give example.

(h) Find the chromatic number of the graph given below :

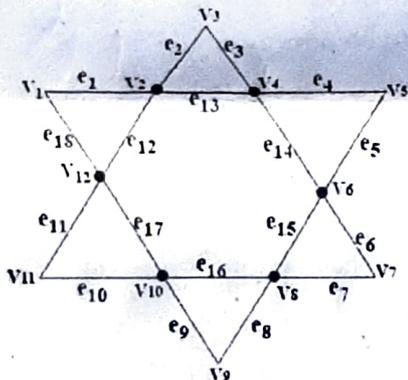


- (i) Define Eccentricity of Vertex.
- (j) What is binary Tree? Give example.

**PART-II**

2. Attempt all the parts : 4x5=20

- (a) Find the terms independent of x in the expansion of  $\left(3x^2 + \frac{1}{3x}\right)^8$ .
- (b) A and B are two sets and U the Universal set such that  $n(U) = 700, n(A) = 200, n(B) = 300$  and  $n(A \cap B) = 100$ . Find  $n(A' \cap B')$ .
- (c) Compute the truth table of the statement  $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$ .
- (d) Explain Euler Graph? Find Euler line of the graph given below :



(e) Prove that the number of vertices in a binary tree is always odd.

**PART-III**

Attempt all the questions by taking Internal choice.

3. (a) Prove by Mathematical Induction method : 6

$$1^2 + 2^2 + 3^2 + 4^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

(b) Find the coefficient of  $x^5$  in the expansion of the product  $(1 + 2x)^6 (1 - x)^7$ . 6

Or

(a) Solve the recurrence relation : 6

$$a_n = 6a_{n-1} - 8a_{n-2} \text{ where } a_0 = 4, a_1 = 10$$

(b) Find the generating function of the Fibonacci Sequence  $\{a_n\}$  defined by 6

$$a_n = a_{n-1} + a_{n-2} \text{ where } a_0 = 0, a_1 = 1$$

4. Prove that the relation R on the set Z of all integers defined by  $(x, y) \in R \Rightarrow x - y$  is divisible by n is an equivalence relation on Z. 6

(b) Let  $A = R - \{2\}$  and  $B = R - \{1\}$  if  $f: A \rightarrow B$  is a mapping defined by  $f(x) = \frac{x-1}{x-2}$  show that f is bijective. 6

Or

(a) If  $f: R \rightarrow R$  and  $g: R \rightarrow R$  are defined by  $f(x) = x + 2$  and  $g(x) = 2x^2 + 5$ , then show that  $f \circ g \neq g \circ f$ . Also  $f \circ g(-1)$  and  $g \circ f\left(\frac{1}{2}\right)$ . 6

(b) Let \* be a binary operation on the set Q of rotational number as  $a * b = a + b - ab$  check whether \* is commutative and associative. 6

5. (a) Prove by using Boolean Algebra properties : 3+3

- (i)  $[a + (a' + b)'] \cdot [a + (b' + c)'] = a$
- (ii)  $(a + b) \cdot (a + b' + c) = a + b \cdot c'$

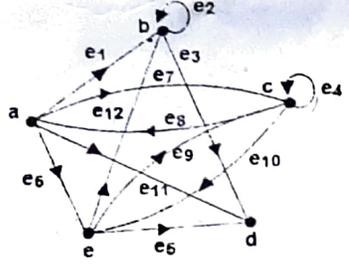
(b) Prove that  $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ . 6

Or

(a) Show by means of a truth table that  $(p \Leftrightarrow q) \equiv (\sim p \vee q) \wedge (\sim q \vee p)$ . 6

- (b) (i) Express the following Boolean function in D.N.F.  
 $[(x_1 + x_2) + (x_2 + x_3)']' + x_2 \cdot x_3$
- (ii) Draw the logic gate diagram of the following boolean function  
 $[(x + y) + (x + z)] \cdot x \cdot y'$  4.2

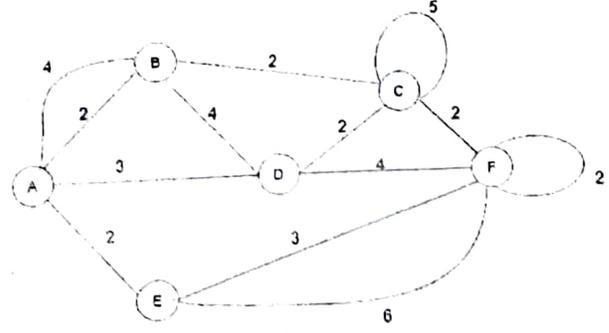
6. (a) Find the Adjacency and incidence matrix of the following graph :



(b) Explain Planar and Non-Planar graph with suitable example. 3+3

Or

(a) Find the Shortest path between the vertex a and f in the following graph by using Dijkstra's Algorithm : 6



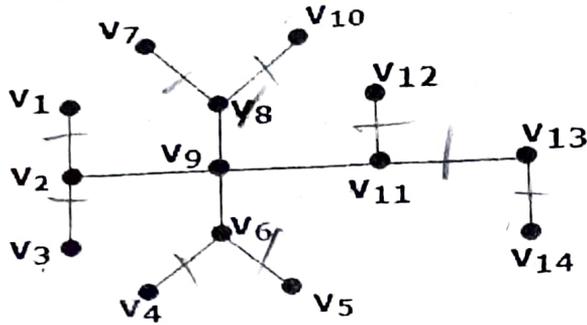
If  $G(V, E)$  be a loop free connected planar graph with  $|V| = v$  vertices  $|E| = e > 2$  edges and  $r$  region, then prove that

(i)  $3r \leq 2e$

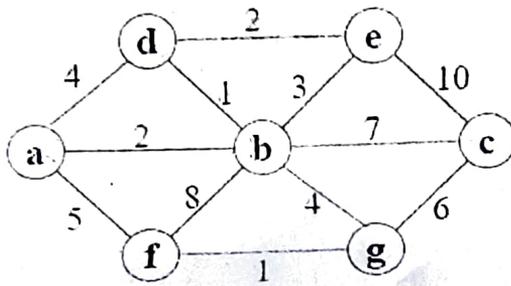
(ii)  $e \leq 3V - 6$

6

7. (a) Find the eccentricity of all the vertices of the given tree below :



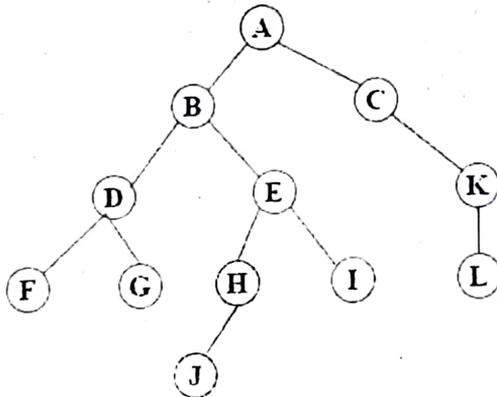
(b) Find a Minimal Spanning tree for a following weighted connected graph by using Prim's algorithm.



Or

(a) Write the Preorder, Inorder and Postorder traversal of the following graph.

6



(b) If  $T$  is binary tree of height  $h$  with  $n$  vertices, then prove that  $h + 1 \leq n \leq 2^{h+1} - 1$ .

6

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